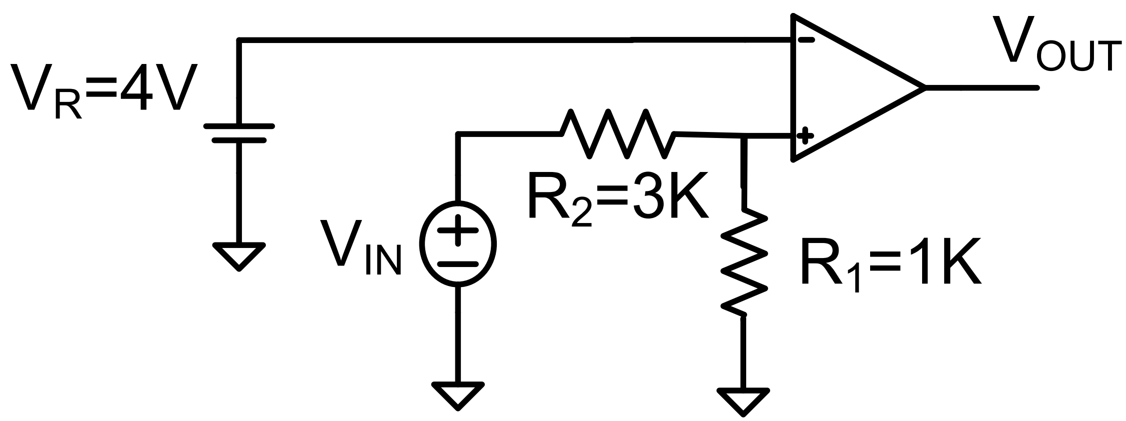
EE 230
Lecture 21

Nonlinear Op Amp Applications

- Nonlinear analysis methods
- Comparators with Hysteresis

Quiz 15

Plot the transfer charactristics of the following circuit. Assume the op amp has V_{SATH}=12V and V_{SATL}=-12V.



And the number is ?

1

3

8

5

4

?

2

6

9

7

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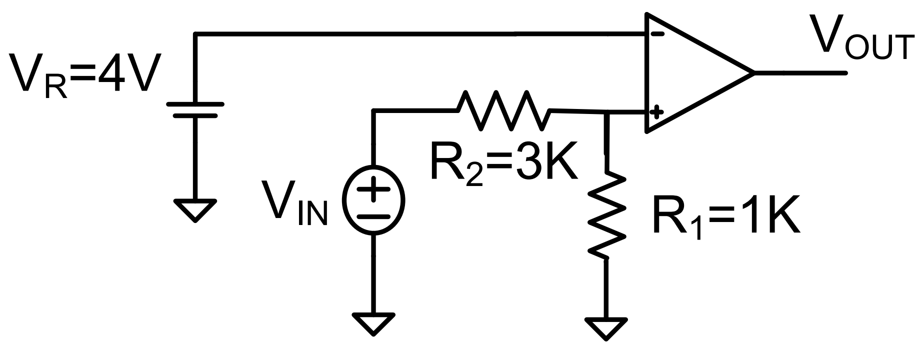
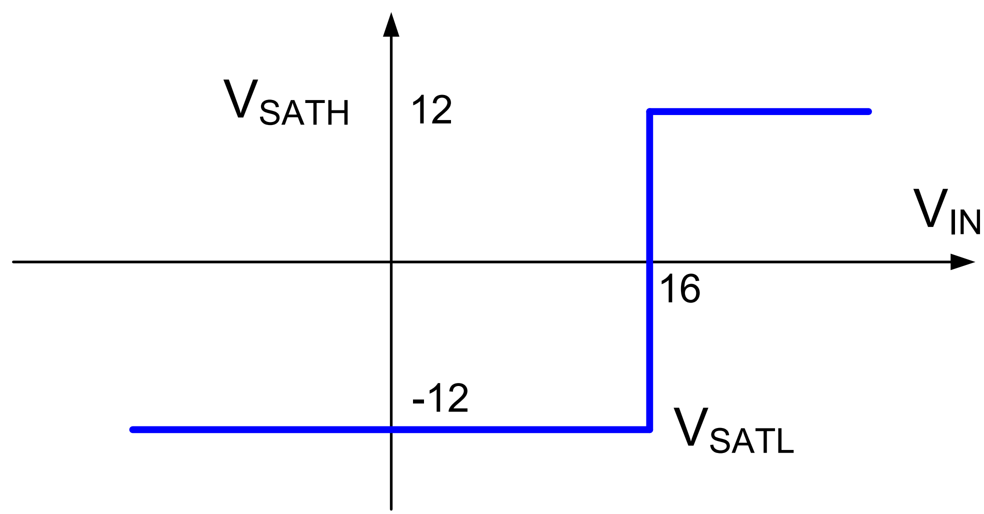


Solution:

$$V^+ = \frac{V_{IN}}{4}$$

$$V_{OUT} = \begin{cases} V_{SATH} & \text{if } V^+ > V_R \\ V_{SATL} & \text{if } V^+ < V_R \end{cases}$$

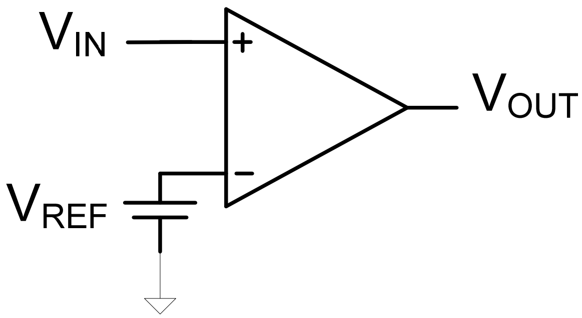
$$V_{OUT} = \begin{cases} V_{SATH} & \text{if } V_{IN} > 16V \\ V_{SATL} & \text{if } V_{IN} < 16V \end{cases}$$



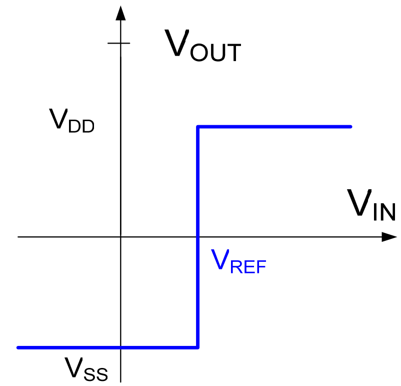
Review from Last Lecture

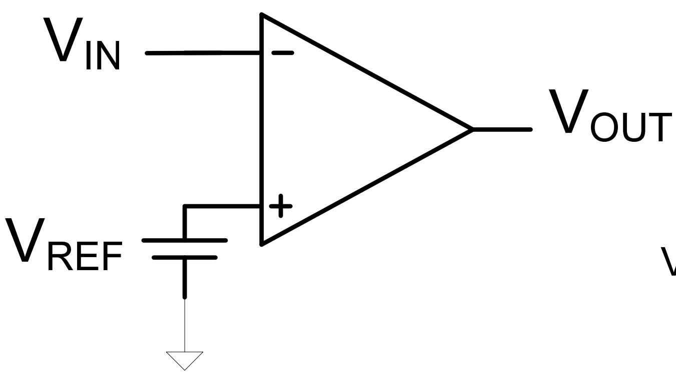
The Comparators

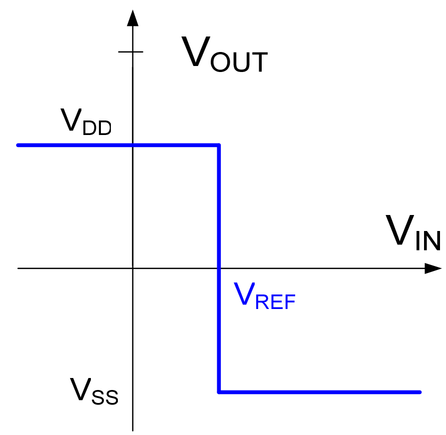
(Assume $V_{SATH}=V_{DD}$, $V_{SATL}=V_{SS}$)



$$V_{OUT} = \begin{cases} V_{DD} & \text{for } V_{IN} > V_{REF} \\ V_{SS} & \text{for } V_{IN} < V_{REF} \end{cases}$$

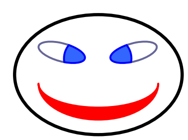






$$V_{OUT} = \begin{cases} V_{DD} & \text{for } V_{IN} < V_{REF} \\ V_{SS} & \text{for } V_{IN} > V_{REF} \end{cases}$$

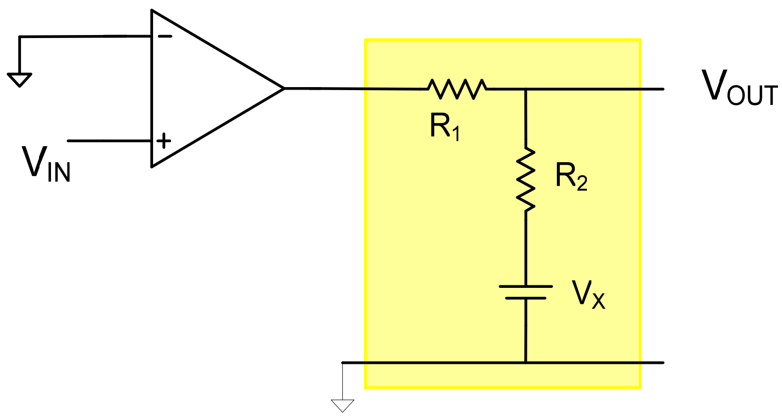
Op Amps make good comparators when operated open-loop

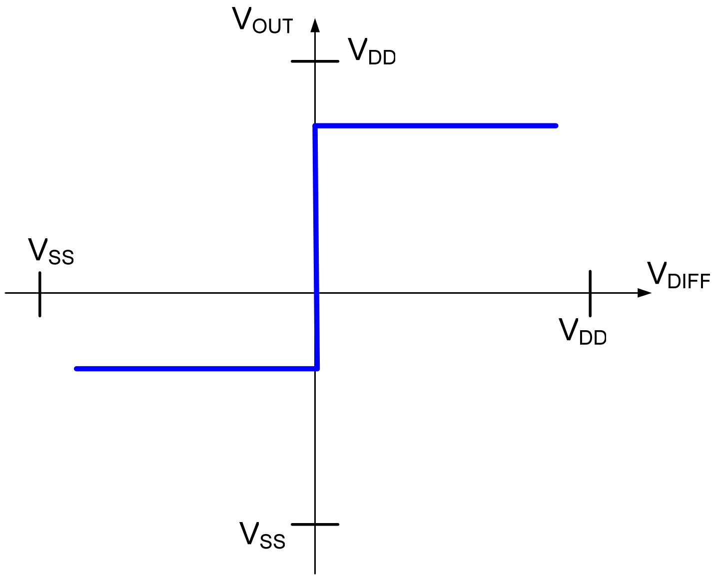


Some ICs are manufactured to serve only as comparators and often have better performance than op amps

Review from Last Lecture

The Op Amp is Highly Nonlinear when Over-Driven





The comparator circuit is also highly nonlinear

Many useful applications of op amps when operating nonlinearly

Many other nonlinear devices exist that are also very useful

Review from Last Lecture

Nonlinear Circuits and Applications

Definition: A circuit is nonlinear if one or more devices in the circuit do not operate linearly

- Superposition can not be used to analyze circuit
- Nonlinear circuit applications
  - Will first consider applications where op amp operates nonlinearly
  - Will then consider other nonlinear devices

Will first discuss the concepts of nonlinear circuits and nonlinear circuit analysis techniques

Review from Last Lecture

Methods of Analysis of Nonlinear Circuits

Will consider three different analysis requirements and techniques for some particularly common classes of nonlinear circuits

1. Circuits with continuously differential devices

   Interested in obtaining transfer characteristics of these circuits or outputs for given input signals

2. Circuits with piecewise continuous devices

   interested in obtaining transfer characteristics of these circuits or outputs for a given input signals

3. Circuits with small-signal inputs that vary around some operating point

   Interested in obtaining relationship between small-signal inputs and the corresponding small-signal outputs. Will assume these circuits operate linearly in some suitably small region around the operating point

Other types of nonlinearities may exist and other types of analysis may be required but we will not attempt to categorize these scenarios in this course

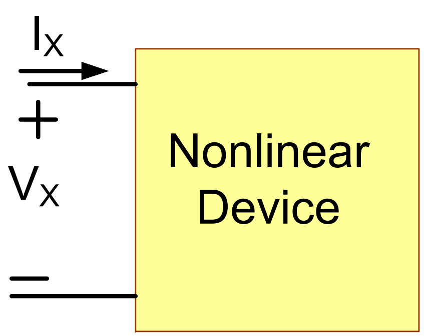
1. Nonlinear circuits with continuously differential devices

Use KVL and KCL for analysis

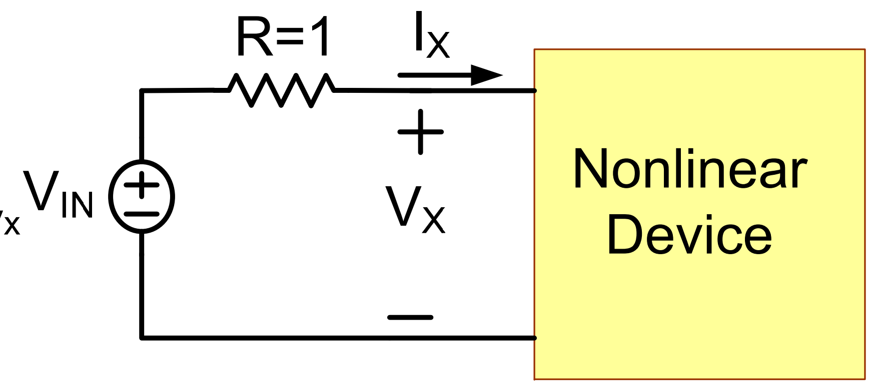
Represent nonlinear models for devices either mathematically or graphically

Solve the resultant set of equations for the variables of interest

Example:



$I_X = 3e^{V_X}$



Circuit with Nonlinear Device $V_X = ?$

$$\left.\begin{aligned} (V_{IN}-V_X)G &= I_X \\ I_X &= 3e^{V_X} \end{aligned}\right\}$$

$$(V_{IN}-V_X) = I_X R$$

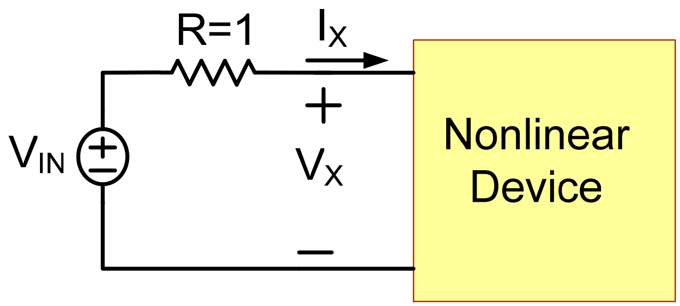
$$V_{IN} = V_X + 3Re^{V_X}$$

Solution relating V_X to V_{IN} is highly nonlinear

Explicit expression for V_X in terms of V_{IN} does not exist !

1. Nonlinear circuits with continuously differential devices

Example:

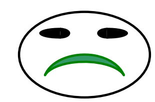


$$V_X = ?$$

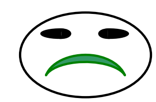
$$V_{IN} = V_X - 3Re^{V_X}$$

Solution relating V_X to V_{IN} is highly nonlinear

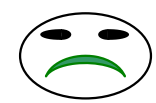
Explicit expression for V_X in terms of V_{IN} does not exist !



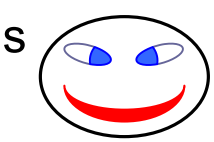
Solution for even modestly more complicated circuits can be really messy



Explicit expressions for V_{IN} or V_X or both are often impossible to obtain



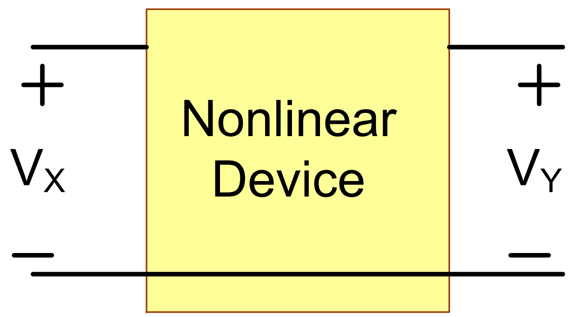
Most useful nonlinear circuits will have reasonably simple final expressions for output variables of interest and a systematic procedure for analyzing these circuits



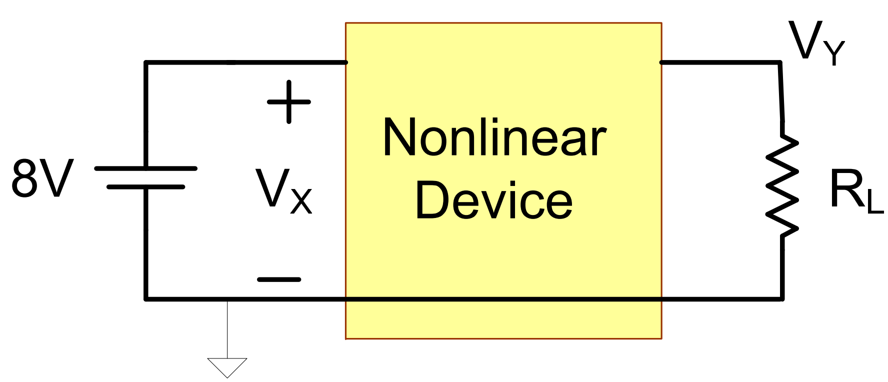
2. Circuits with piecewise continuous devices

1. Guess region of operation
2. Solve resultant circuit using the previous method
3. Verify region of operation is valid
4. Repeat the previous 3 steps as often as necessary until region of operation is verified

Example:



$$V_Y = \begin{cases} 0 & V_X < 1V \\ 4(V_X - 1) & V_X > 1V \end{cases}$$



$V_Y = ?$

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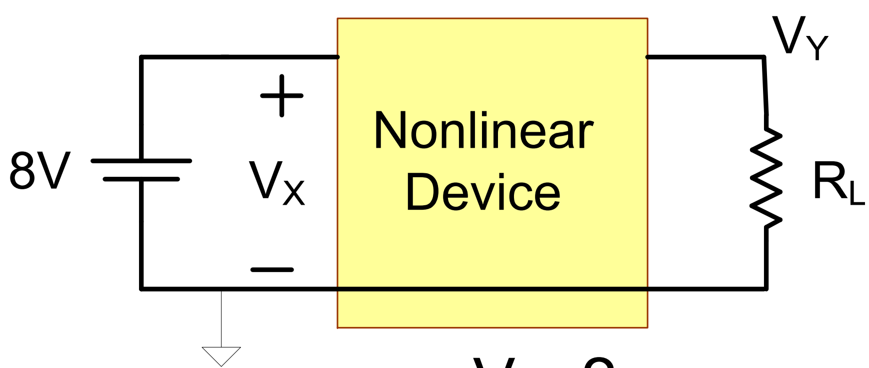


V_Y=?

$$V_Y=\begin{cases}0 & V_X<1V \\ 4(V_X-1) & V_X>1V\end{cases}$$

Region 1

Region 2

Guess Region 1

V_Y=0V

verify V_X <? 1V

V_X=8V

Verification fails (solution not valid)

Guess Region 2

V_Y=4(8-1)=28V

verify V_X >? 1V

V_X=8V

V_X =8V > 1V

Verification passes (solution valid)

$$\boxed{V_Y = 28V}$$

3. Circuits with small-signal inputs that vary around some operating point

Determine the operating point (using method 1 or 2 discussed above after all small signal independent inputs are set to 0)

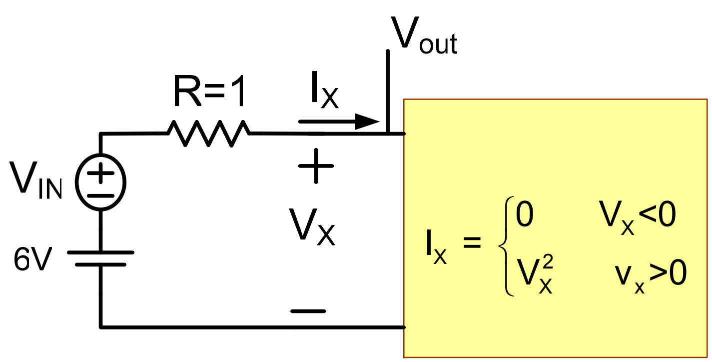
Develop small signal (linear) model for all devices in the region of interest (around the operating point or “Q-point”)

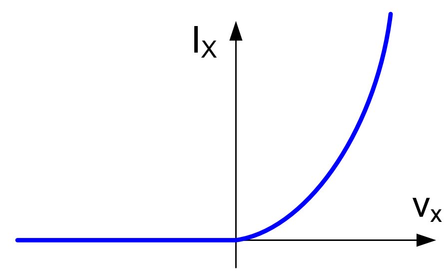
Create small signal equivalent circuit by replacing <u>all</u> devices with small-signal equivalent

Solve the resultant small-signal (linear) circuit
Can use KCL, KVL, and other linear analysis tools such as superposition, voltage and current divider equations, Thevenin and Norton equivalence

Determine boundary of region where small signal analysis is valid

Example:

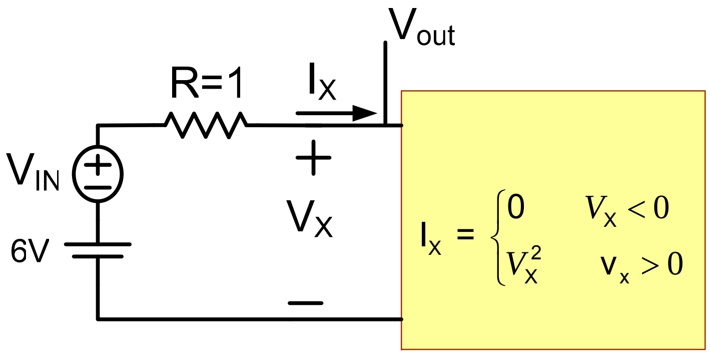


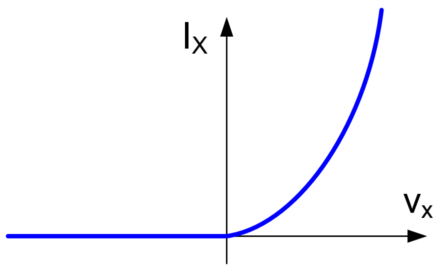


V_{IN} is a small signal, much smaller than 6V

3. Circuits with small-signal inputs that vary around some operating point

Example:





V_{IN} is a small signal, much smaller than 6V

Will go through the mechanics of this process at this time but will develop and justify the steps later in the course !

This is a very useful process that is used widely in the electronics field and in many other fields as well

Students will not be expected to do this type of analysis until the process is formally developed

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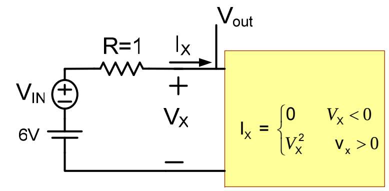
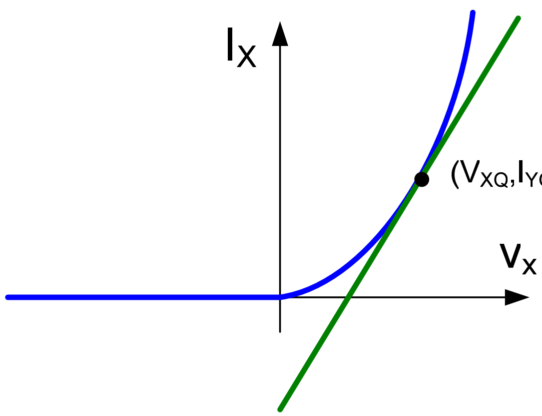
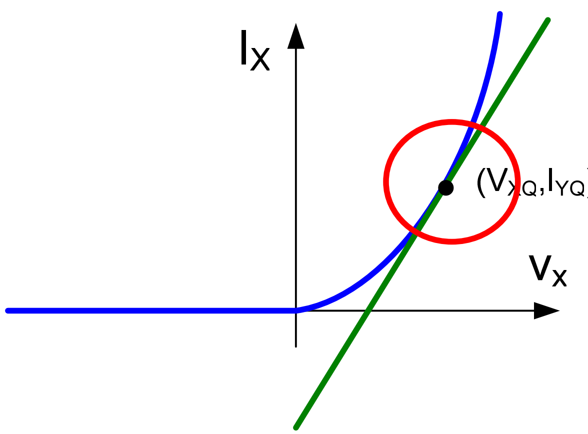


Example:



Will linearize the circuit at the operating point (Q-point (V_{XQ}, I_{XQ}))



Linear region shown

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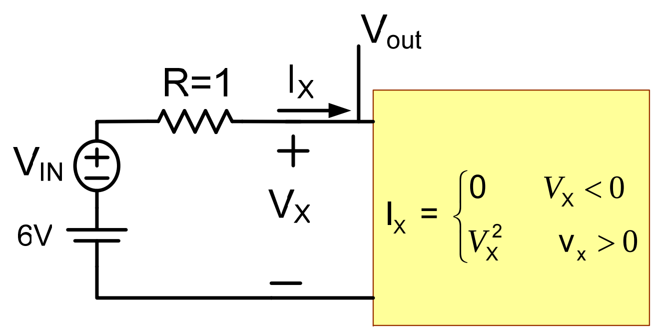
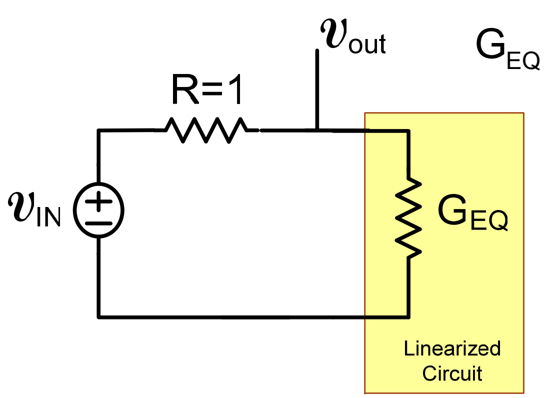


Example:



$$G_{EQ} = \left.\frac{\partial I_X}{\partial V_X}\right|_{(V_{XQ}, I_{XQ})}$$

Small-signal equivalent circuit

Solution of Small-Signal Linear Circuit

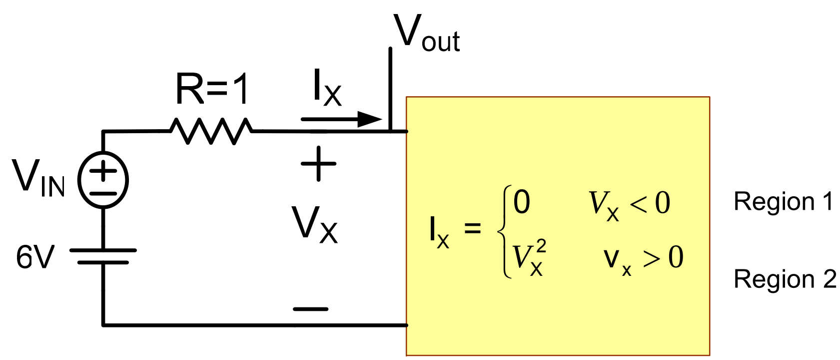
$$v_{OUT} = \frac{1}{1+RG_{EQ}} v_{IN}$$

Overall output if required

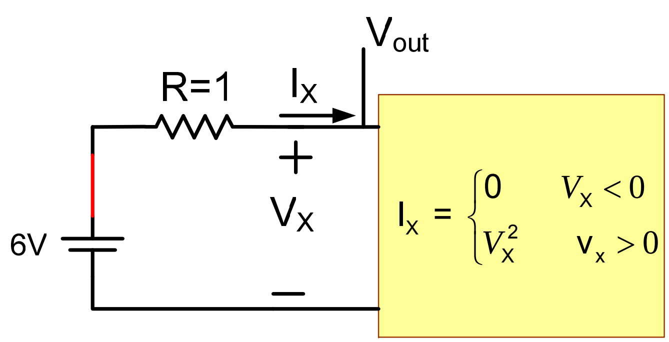
$$V_{OUT} = V_{OUTQ} + v_{OUT} = V_{XQ} + \frac{1}{1+RG_{EQ}} v_{IN}$$

3. Circuits with small-signal inputs that vary around some operating point

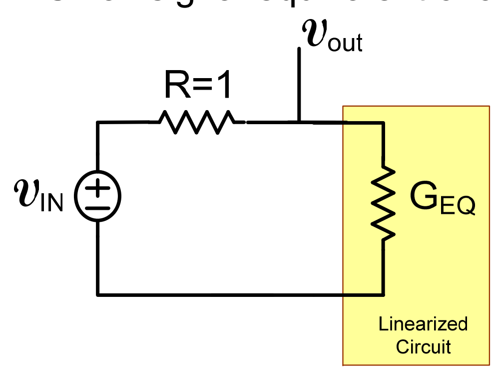
Example:



Circuit with small-signal sources zeroed out



Small signal equivalent circuit



Guess Region 2 (must verify $V_x>0$)

$$\left.\begin{aligned} 6 &= I_X R + V_X \\ I_X &= V_X^2 \end{aligned}\right\} \quad V_X^2 R + V_X - 6 = 0$$

with R=1, obtain the solution

$V_X = 2$ $\qquad (V_{XQ}, I_{XQ}) = (2,4)$

veriify $V_X > 0$

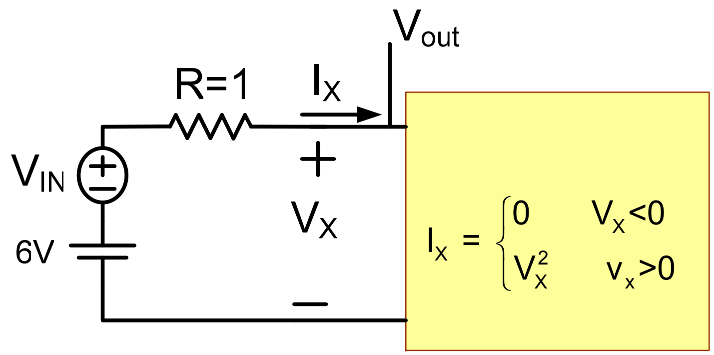
$$G_{EQ} = \left.\frac{\partial I_X}{\partial V_X}\right|_{(V_{XQ}, I_{XQ})}$$

$$G_{EQ} = \left.\frac{\partial V_X^2}{\partial V_X}\right|_{(V_{XQ}, I_{XQ})} = 2V_X\Big|_{(2,4)} = 4$$

$$v_{OUT} = \frac{1}{1+RG_{EQ}} v_{IN} = \frac{1}{1+1\bullet 4} v_{IN} = 0.2 \bullet v_{IN}$$

3. Circuits with small-signal inputs that vary around some operating point

Example:



Region 1

Region 2

$$v_{OUT} = 0.2 \bullet v_{IN}$$

$$V_{OUT} = V_{XQ} + \frac{1}{1+RG_{EQ}} v_{IN} = 2 + 0.2 \bullet v_{IN}$$

If V_{IN}=0.1 sinωt,

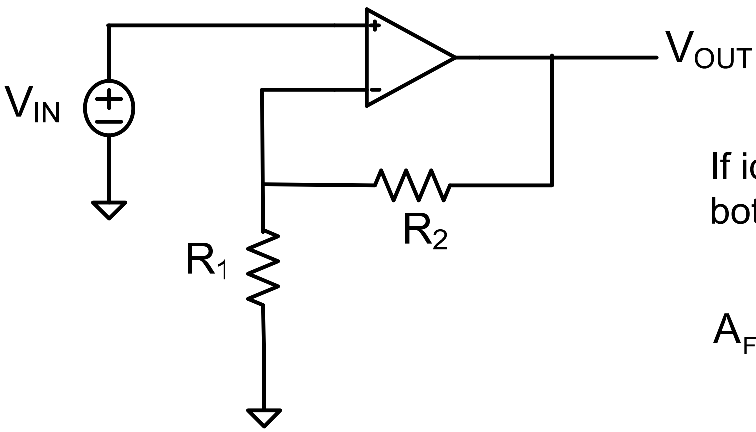
$$v_{OUT} = .02 \sin \omega t$$

$$V_{OUT} = 2 + .02 \sin\omega t$$

Nonlinear Circuits

Will now investigate several nonlinear circuits

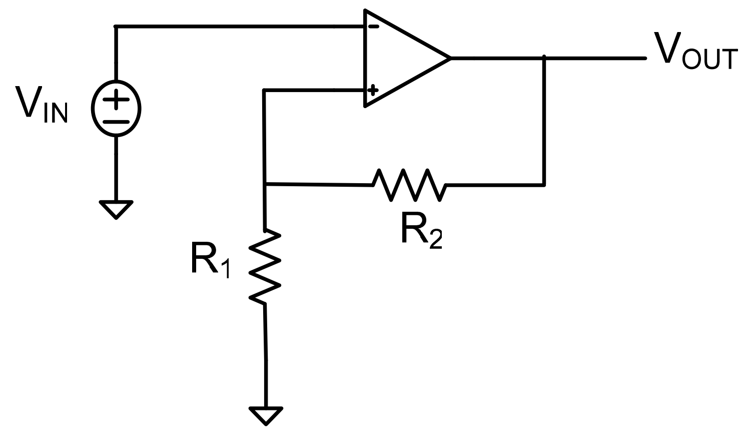
Before trashing the “bad” circuit which we saw was unstable, lets see if it has any useful properties!



Usually the good circuit

If ideal op amps both have gain

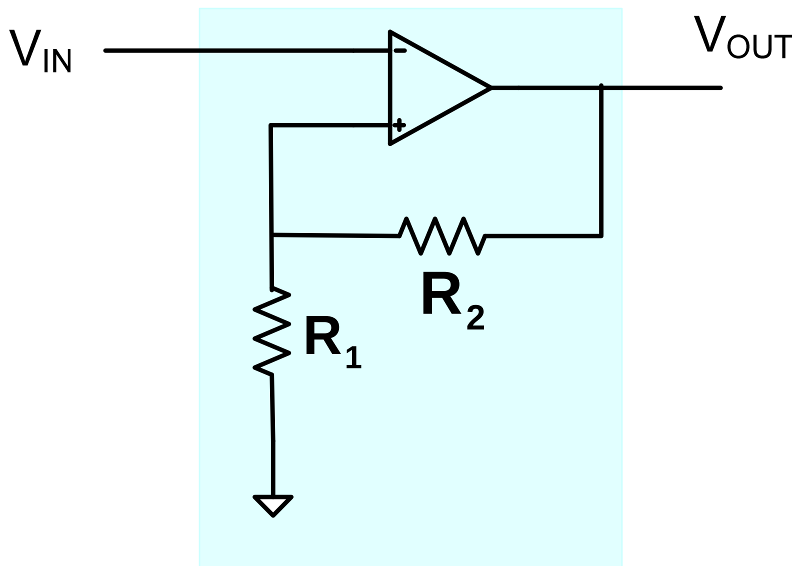
$$A_{FB}=1+\frac{R_2}{R_1}$$



Usually the bad circuit

This circuit is unstable !

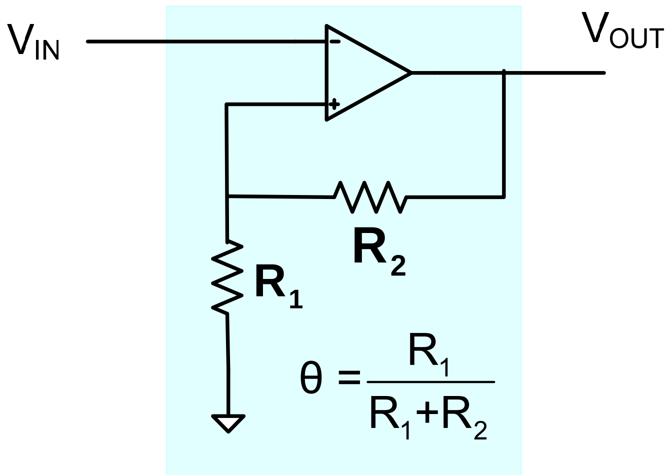
Consider this circuit

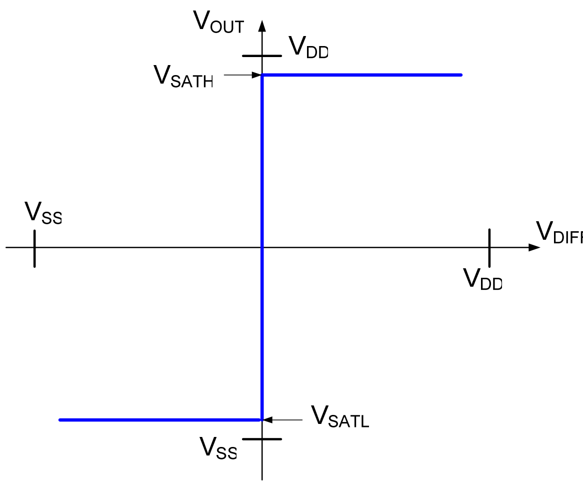


This circuit is an unstable noninverting amplifier and is usually not useful as an amplifier

But what does the circuit really do?

Consider this circuit





$$V_{OUT} = \begin{cases} V_{SATH} & V_{DIFF} > V_{SATH}/A_0 \\ A_0 V_{IN} & V_{SATL}/A_0 < V_{DIFF} < V_{SATH}/A_0 \\ V_{SATL} & V_{DIFF} < V_{SATL}/A_0 \end{cases}$$

Region 1

Region 2

Region 3

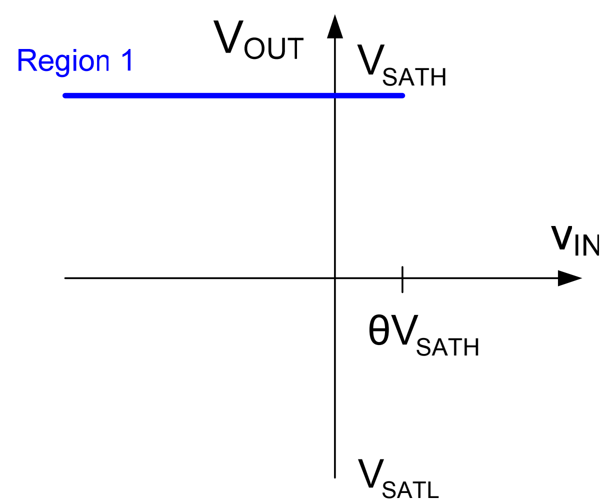
Assume in Region 1 (must verify)

$$V_{OUT} = V_{SATH}$$

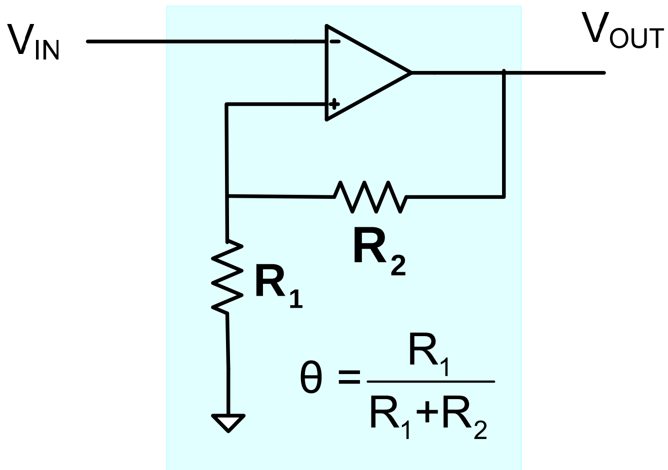
Valid for

$$\theta V_{SATH} - V_{IN} > \frac{V_{SATH}}{A_0}$$

$$\theta V_{SATH} > V_{IN}$$



Consider this circuit



$$V_{OUT} = \begin{cases} V_{SATH} & V_{DIFF} > V_{SATH}/A_0 & \text{Region 1} \\ A_0 V_{IN} & V_{SATL}/A_0 < V_{DIFF} < V_{SATH}/A_0 & \text{Region 2} \\ V_{SATL} & V_{DIFF} < V_{SATL}/A_0 & \text{Region 3} \end{cases}$$

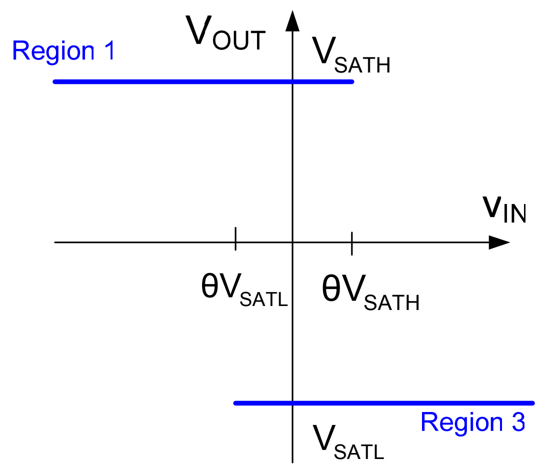
Assume in Region 3 (must verify)

$$V_{OUT} = V_{SATL}$$

Valid for

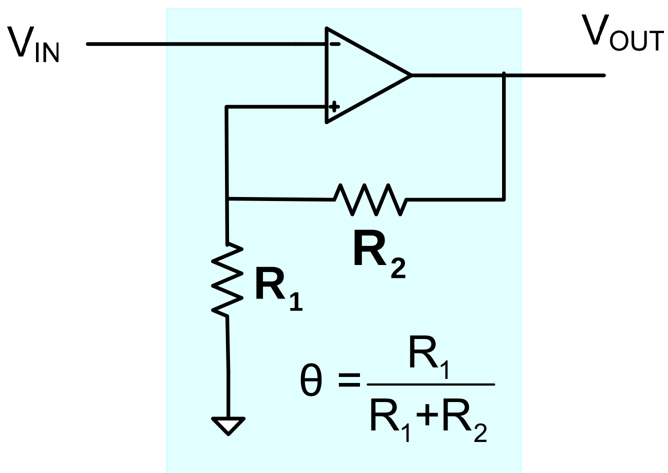
$$\theta V_{OUT} - V_{IN} < \frac{V_{SATL}}{A_0}$$

$$\theta V_{SATL} < V_{IN}$$



Note two-valued for a range of V_{IN}

Consider this circuit



$$V_{OUT} = \begin{cases} V_{SATH} & V_{DIFF} > V_{SATH}/A_0 \\ A_0 V_{IN} & V_{SATL}/A_0 < V_{DIFF} < V_{SATH}/A_0 \\ V_{SATL} & V_{DIFF} < V_{SATL}/A_0 \end{cases}$$

Region 1

Region 2

Region 3

Assume in Region 2 (must verify)

$$\left.\begin{aligned} V_{DIFF} &= \theta V_{OUT} - V_{IN} \\ V_{OUT} &= A_0 V_{DIFF} \end{aligned}\right\}$$

$$V_{OUT} = \left(\frac{1}{\theta - A_0^{-1}}\right) V_{IN}$$

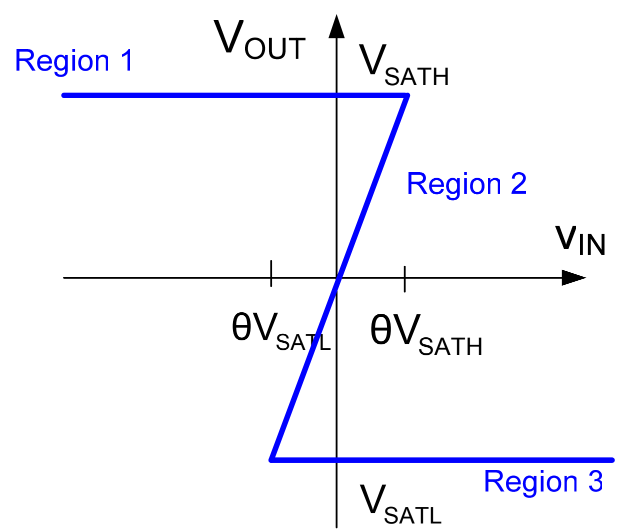
$$V_{OUT} = \left(\frac{1}{\theta}\right) V_{IN}$$

Valid for

$$\frac{V_{SATL}}{A_0} < \theta V_{OUT} - V_{IN} < \frac{V_{SATH}}{A_0}$$

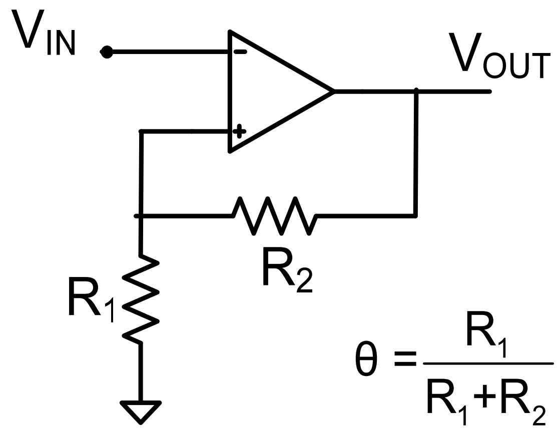
(must use exact value for V_{OUT} to avoid degenerate conditions)

$$\theta V_{SATL} < V_{IN} < \theta V_{SATH}$$

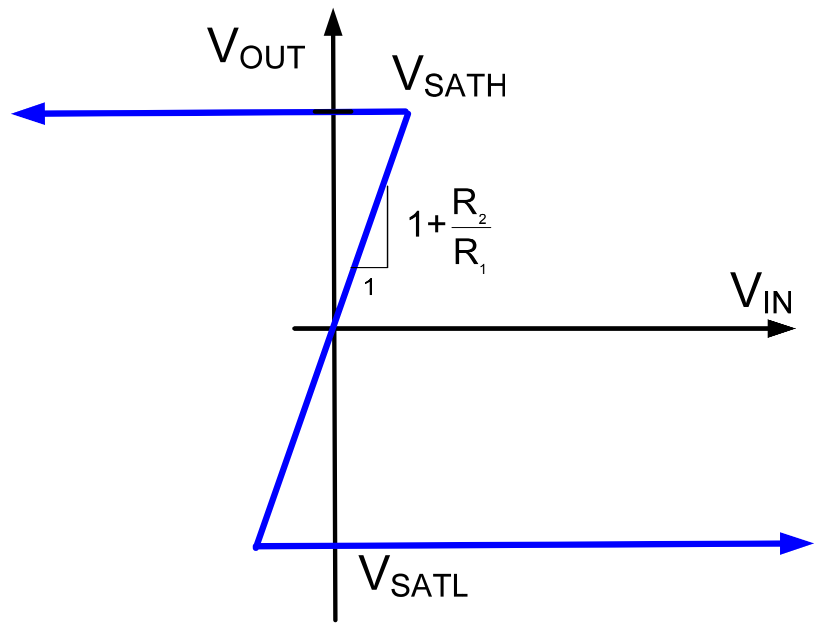


Note tripple-valued for a range of V_{IN}

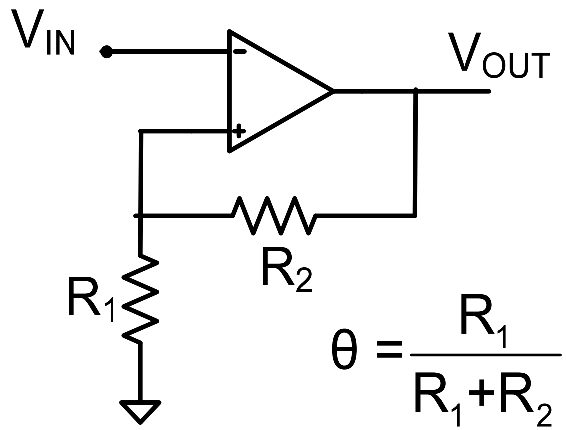
Comparator with Hysteresis

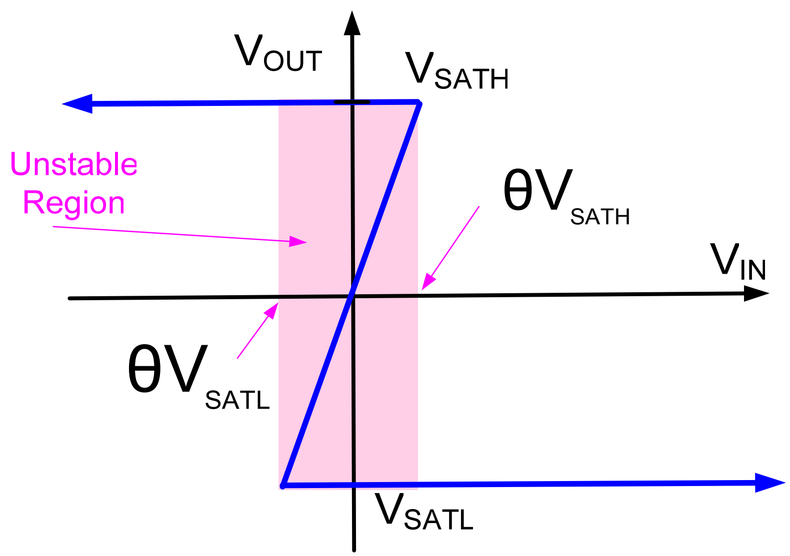


$$\theta = \frac{R_1}{R_1 + R_2}$$

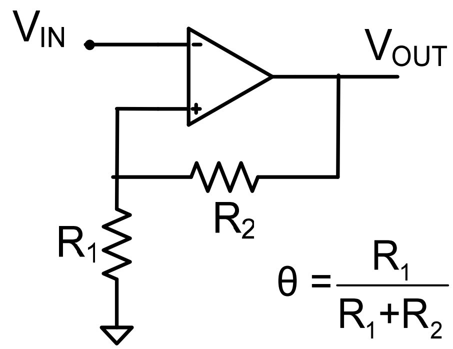


Comparator with Hysteresis



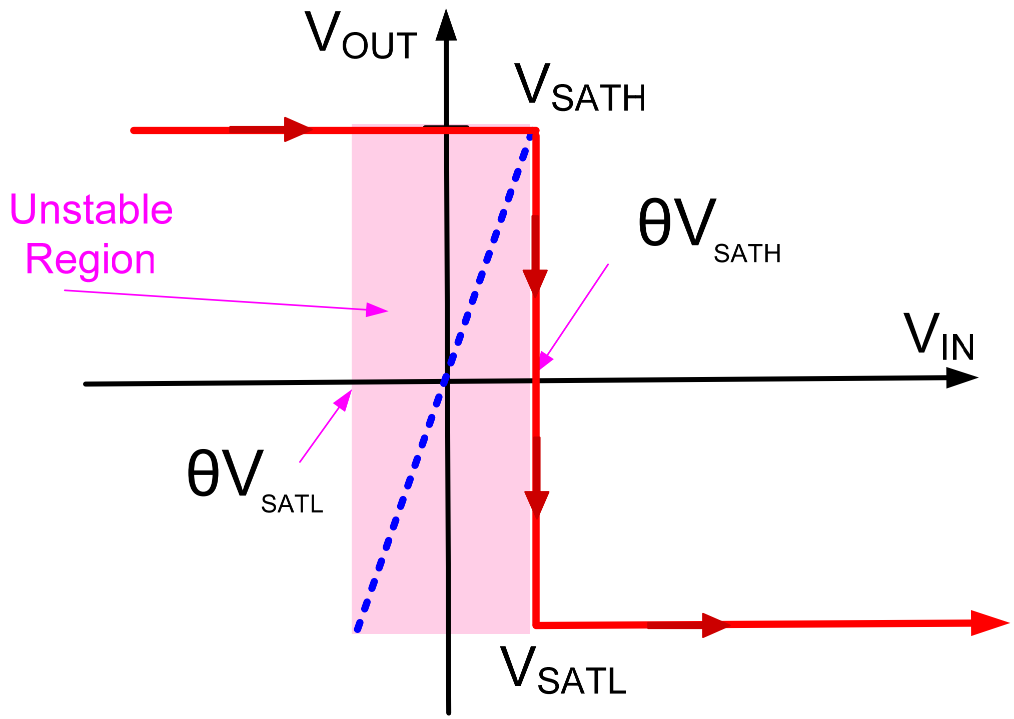


Comparator with Hysteresis

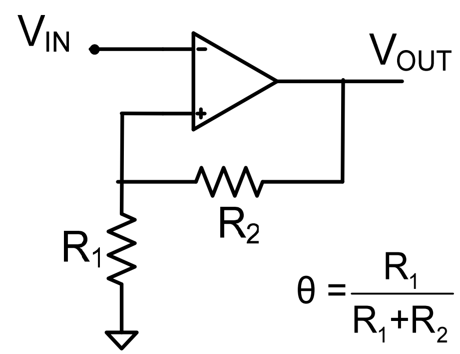


$$\theta = \frac{R_1}{R_1+R_2}$$

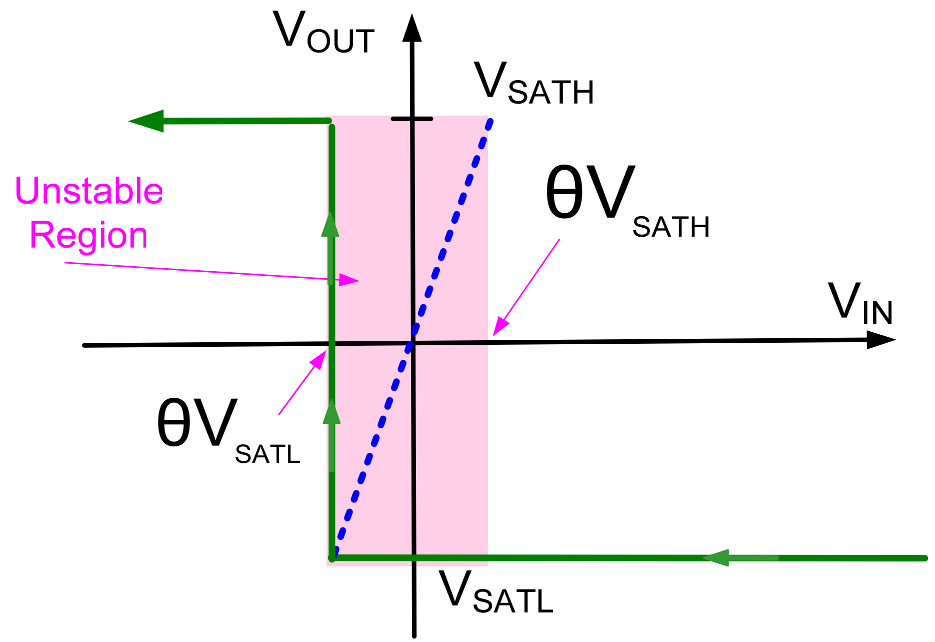
If unstable region is entered from the left



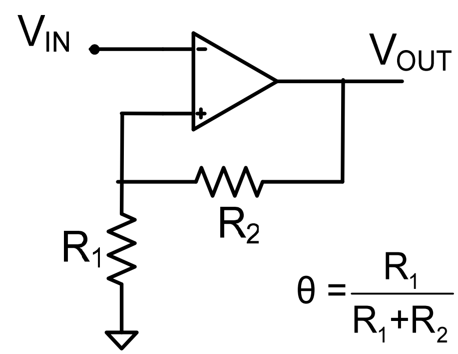
Comparator with Hysteresis



If unstable region is entered from the right

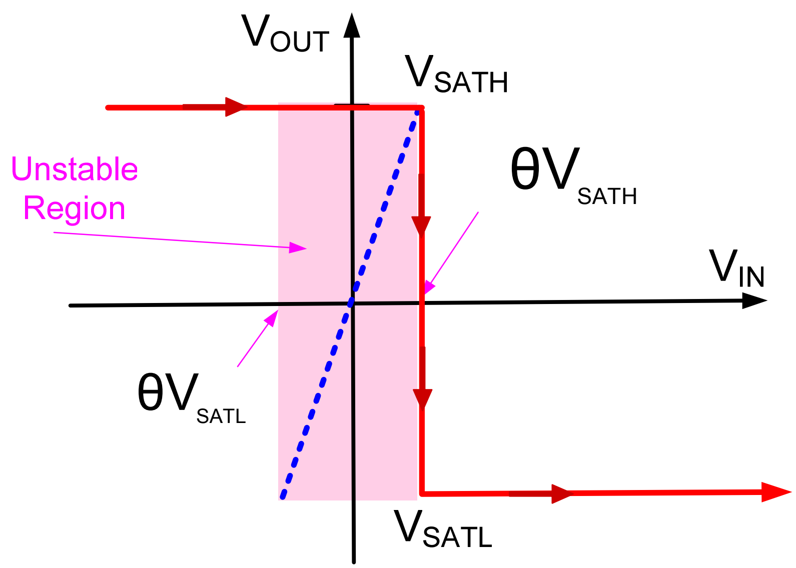


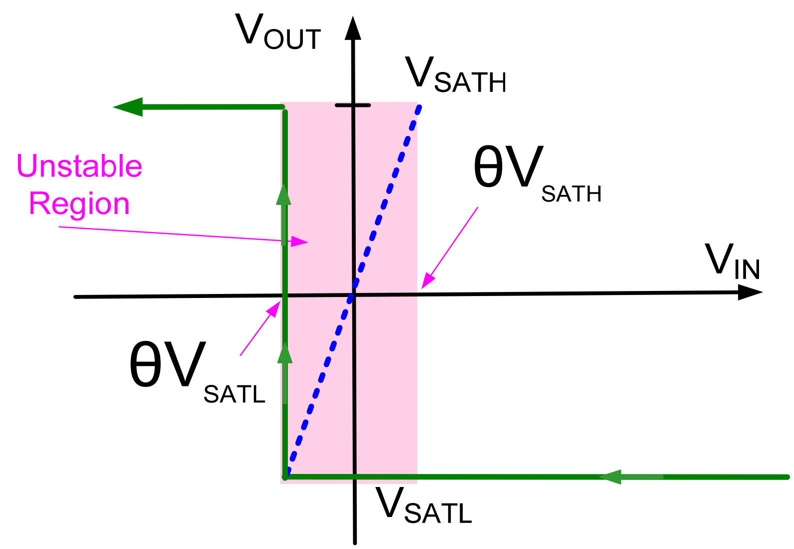
Comparator with Hysteresis



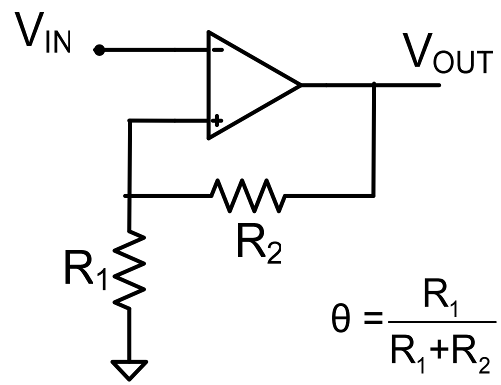
$$\theta = \frac{R_1}{R_1+R_2}$$

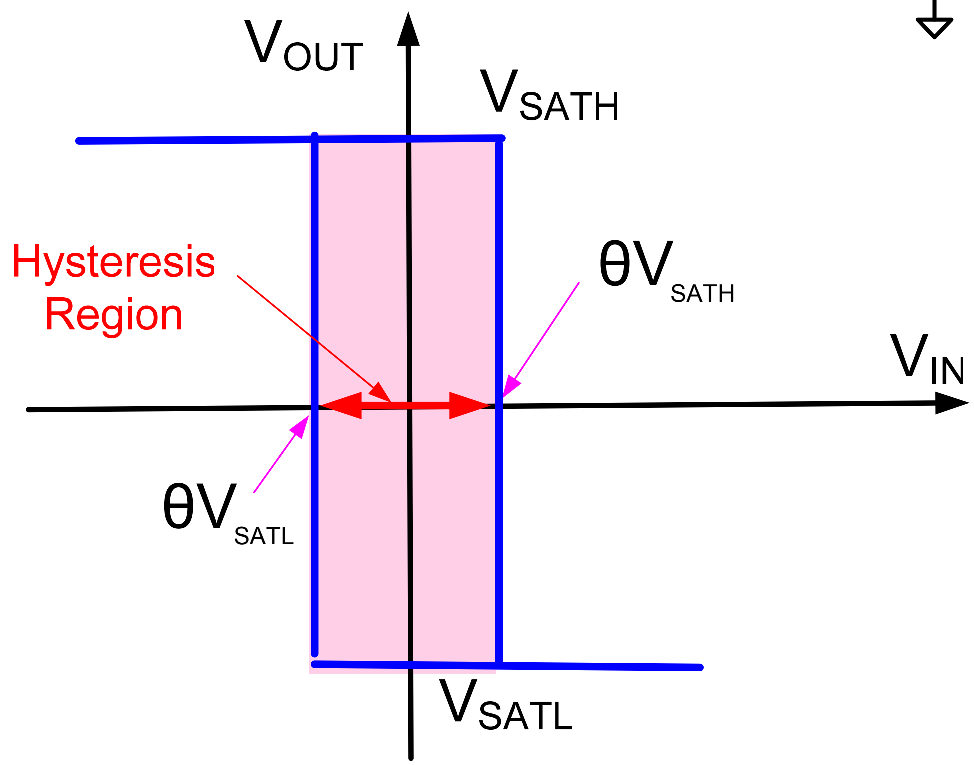
If unstable region is entered from the left or right



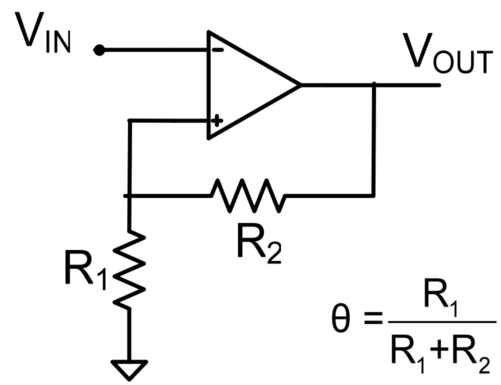


Comparator with Hysteresis

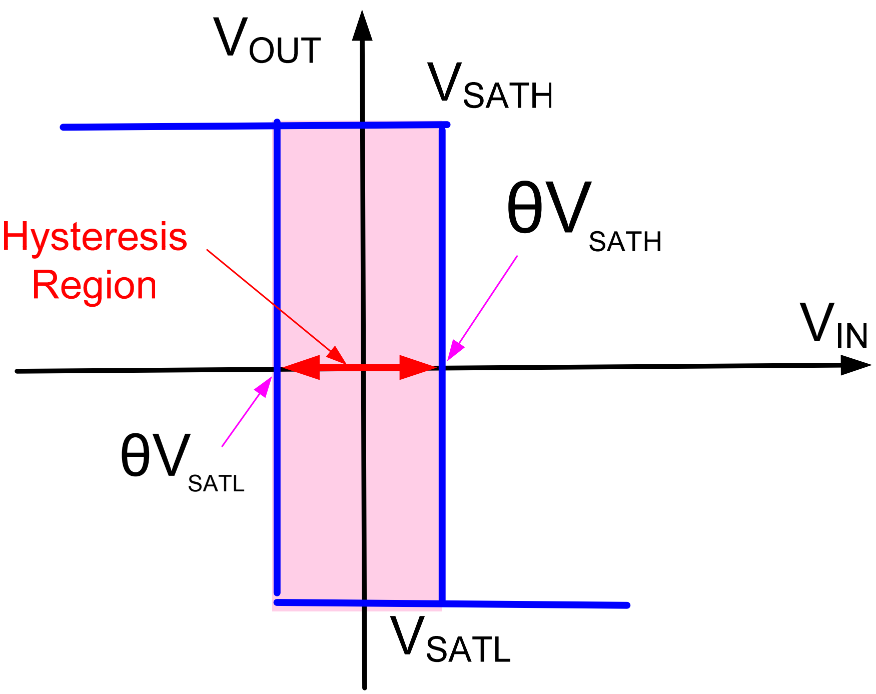




Comparator with Hysteresis



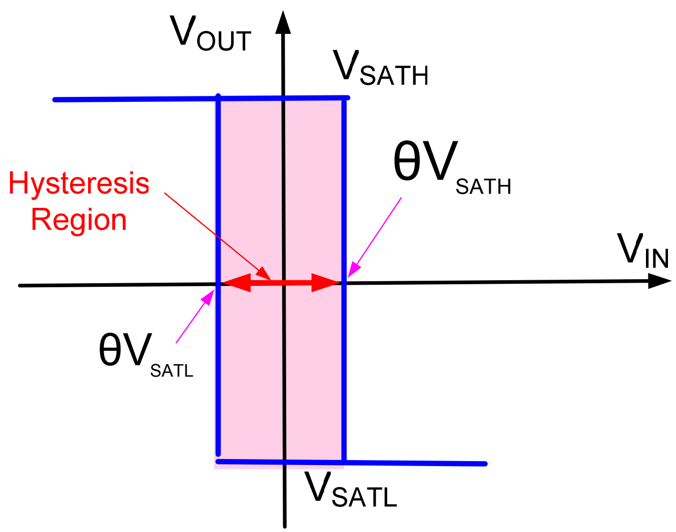
$$\theta = \frac{R_1}{R_1 + R_2}$$

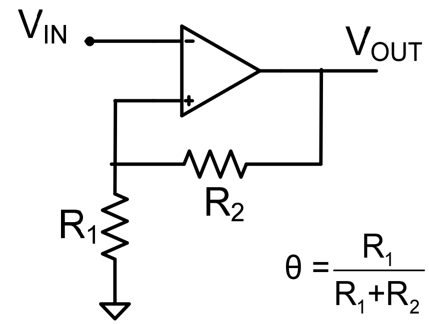


Width of Hysteresis Region

$$\theta\left(V_{SATH} - V_{SATL}\right)$$

Comparator with Hysteresis





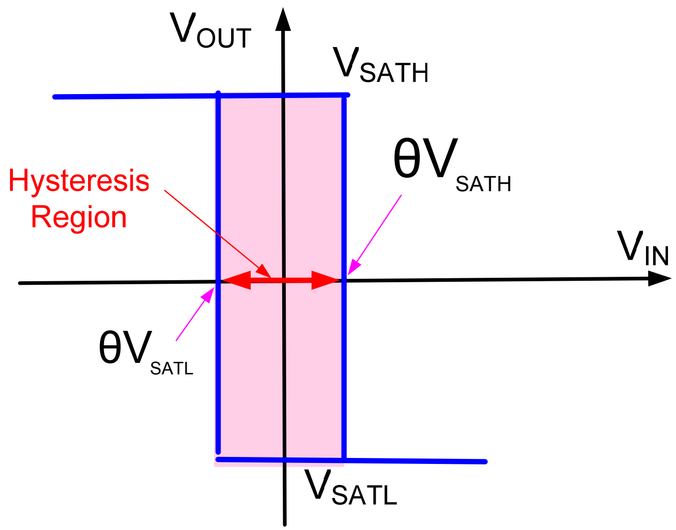
$$\theta = \frac{R_1}{R_1+R_2}$$

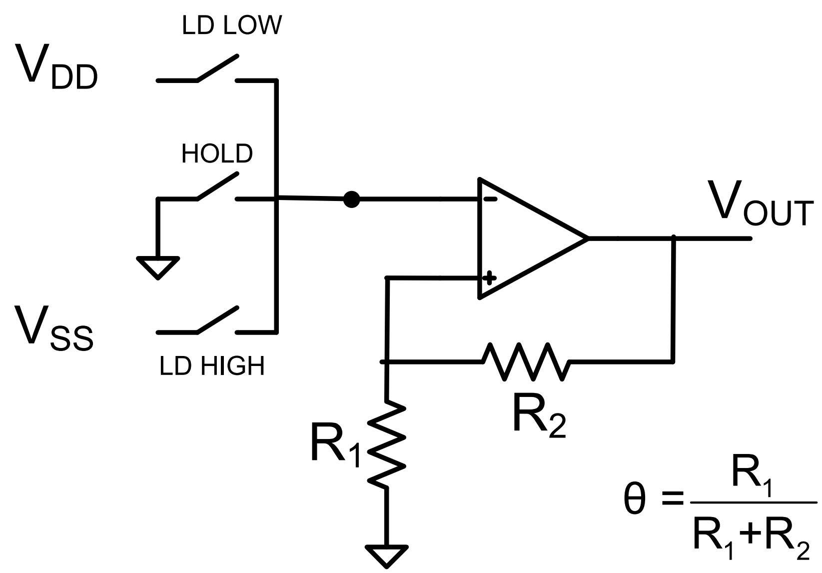
Width of Hysteresis Region

$$\theta\left(V_{SATH} - V_{SATL}\right)$$

- Bistable if V_{IN} is in the hysteresis window
- Often θ is very small
- Widely used in control applications
- Serves as a memory if V_{IN}=0

Comparator as Boolean Memory

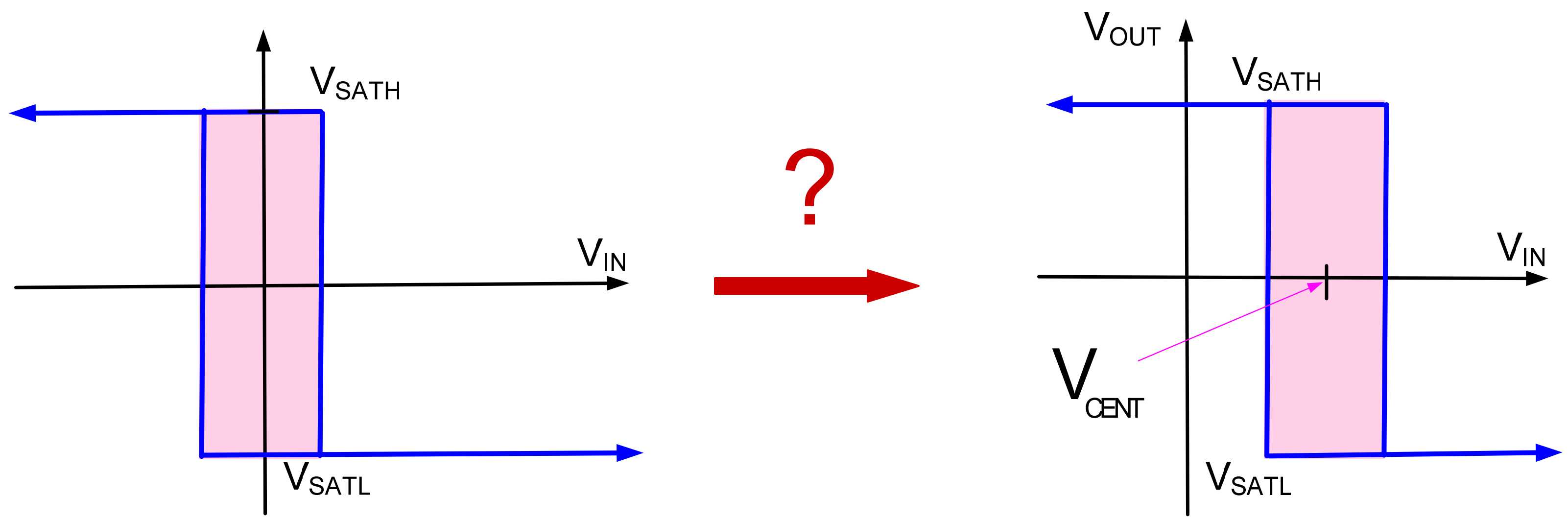




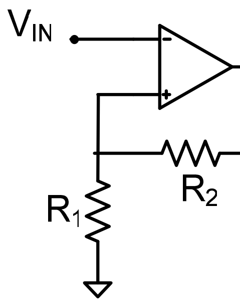
$$\theta = \frac{R_1}{R_1+R_2}$$

- Not cost-competitive with other memory structures
- May be useful, though, in limited applications

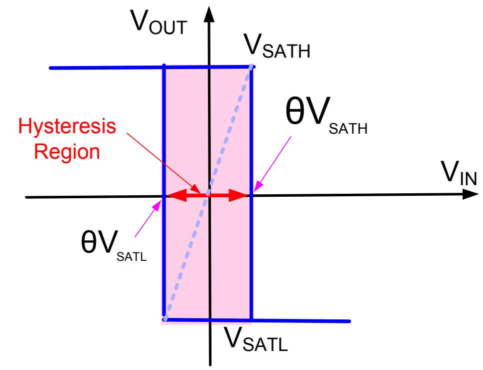
Movement of Hysteresis Loop

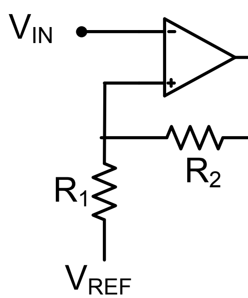


Modifications of Comparator with Hysteresis

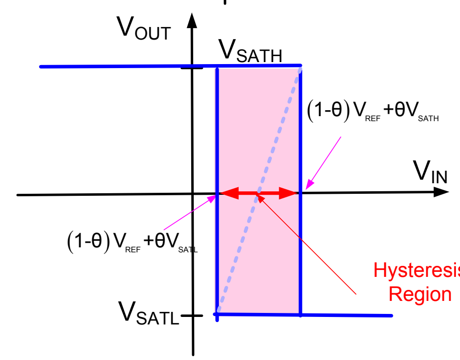


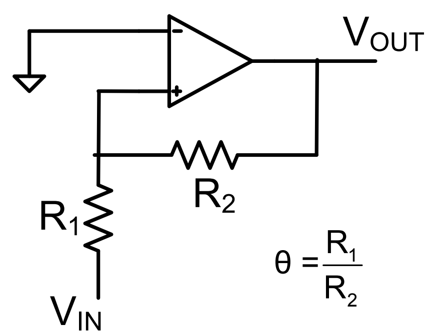
$\theta = \frac{R_1}{R_1+R_2}$





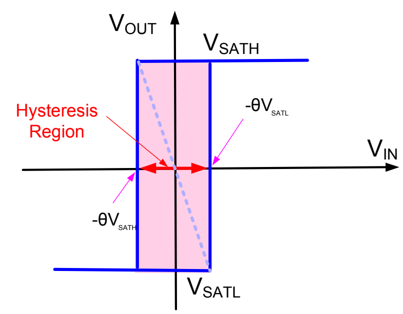
$\theta = \frac{R_1}{R_1+R_2}$





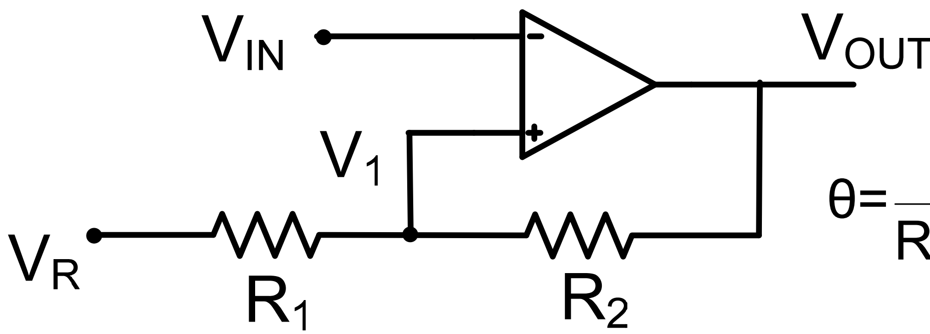
$\theta = \frac{R_1}{R_2}$

Note this is the basic inverting amplifier with op amp terminals interchanged



Many other ways to control position and size of hysteresis window

Movement of Hysteresis Loop



$$\theta=\frac{R_1}{R_1+R_2} \qquad 1-\theta=\frac{R_2}{R_1+R_2}$$

Consider adding a dc voltage V_R

Edges of hysteresis loop determined by condition where $V^{+}=V^{-}$

For this circuit that is where

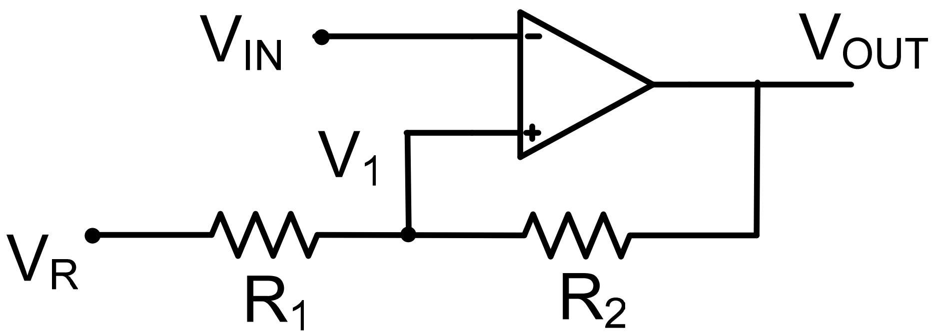
$$V_1=V_{IN}$$

It follows from the 2-input voltage divider equation that

$$V_1=\theta V_{OUT}+\left(1-\theta\right)V_R$$

Substituting the second equation into the first, the edges of the hysteresis loop can be obtained by solving for the two possible values of V_{OUT}, V_{SATH} and V_{SATL}

Movement of Hysteresis Loop

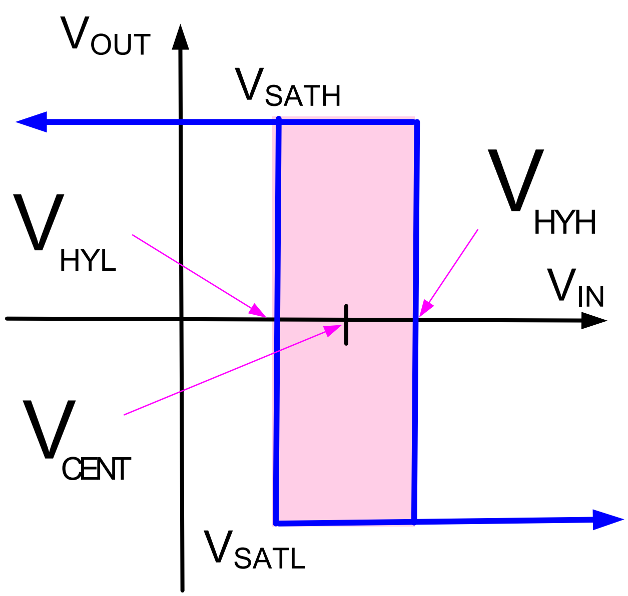


Shifted Inverting Comparator with Hysteresis

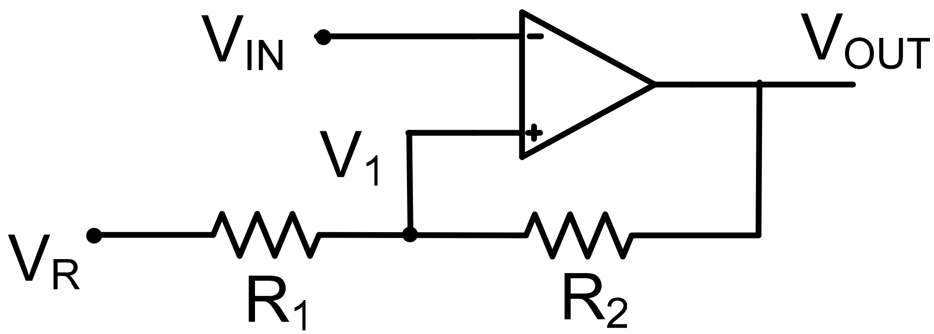
$$\theta=\frac{R_1}{R_1+R_2} \qquad 1\text{-}\theta=\frac{R_2}{R_1+R_2}$$

$$V_{SATH}\cong V_{DD} \qquad V_{SATL}\cong V_{SS}$$

$$V_{IN}=\theta V_{OUT}+(1-\theta)V_R \quad \Rightarrow \quad \begin{cases} V_{HYH}=\theta V_{SATH}+(1-\theta)V_R \\ V_{HYL}=\theta V_{SATL}+(1-\theta)V_R \end{cases}$$



Movement of Hysteresis Loop

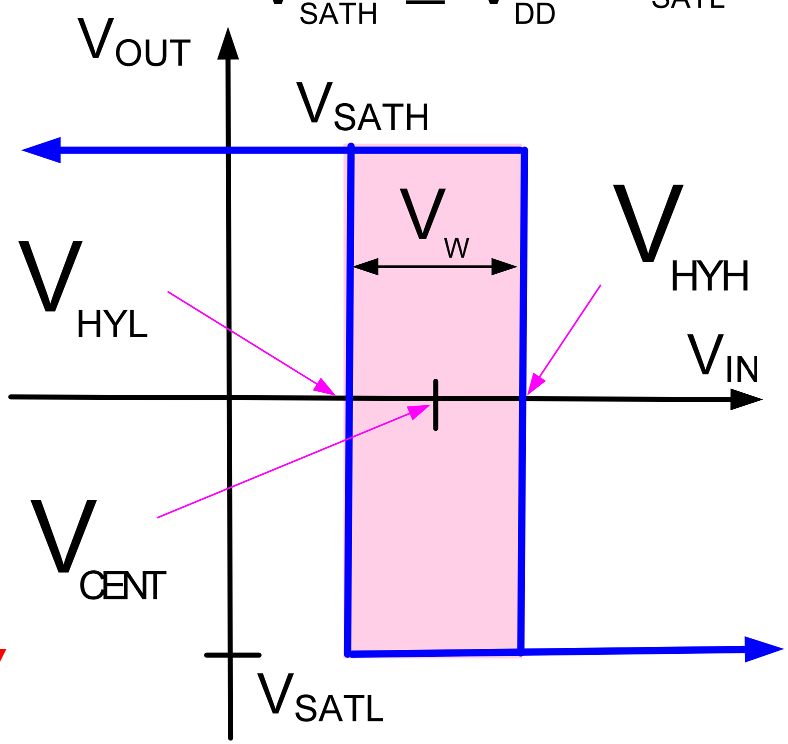


$$\theta=\frac{R_1}{R_1+R_2} \qquad 1-\theta=\frac{R_2}{R_1+R_2}$$

$$V_{HYH}=\theta V_{SATH}+(1-\theta)V_R$$

$$V_{HYL}=\theta V_{SATL}+(1-\theta)V_R$$

$$V_{SATH}\cong V_{DD} \qquad V_{SATL}\cong V_{SS}$$



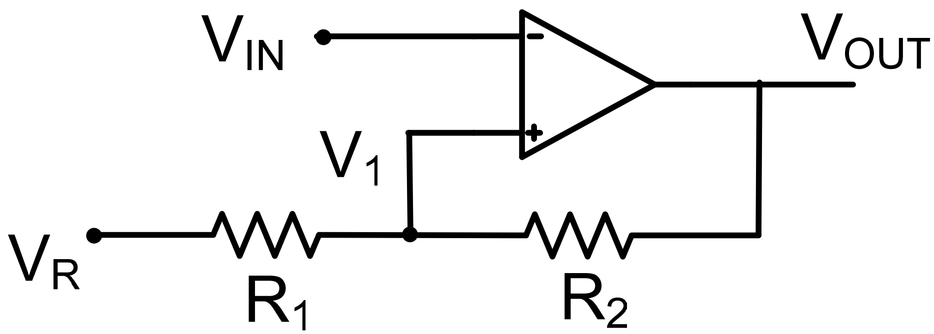
$$V_W=V_{HYH}-V_{HYL}$$

$$V_W=\theta\left(V_{SATH}-V_{SATL}\right)$$

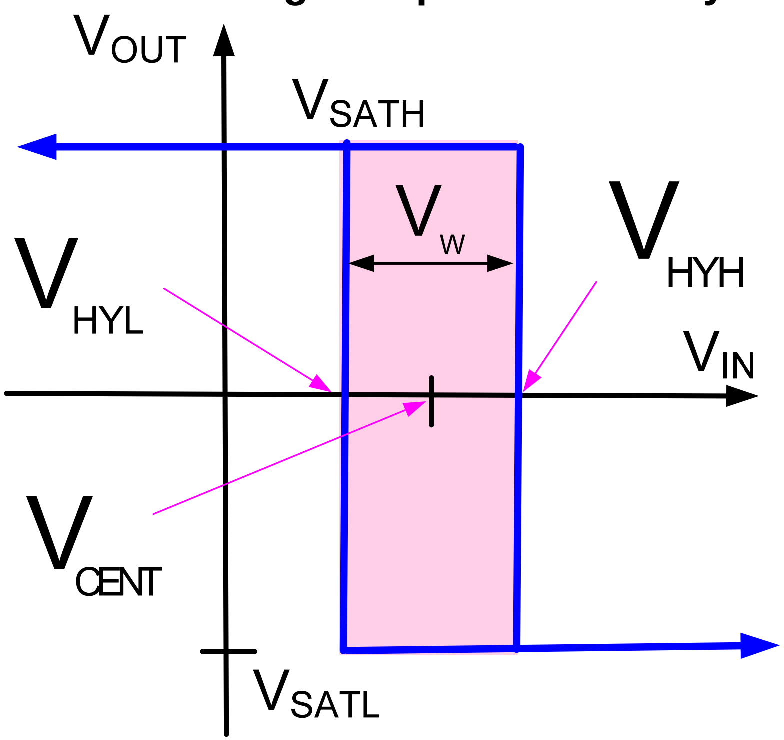
$$V_{CENT}=\frac{V_{HYH}+V_{HYL}}{2}$$

$$V_{CENT}=\left(\frac{\theta\left(V_{SATH}+V_{SATL}\right)}{2}\right)+(1-\theta)V_R$$

Movement of Hysteresis Loop



Shifted Inverting Comparator with Hysteresis



$$\theta = \frac{R_1}{R_1 + R_2}$$

$$V_W = \theta\left(V_{SATH} - V_{SATL}\right)$$

$$V_{CENT} = \left(\frac{\theta\left(V_{SATH} + V_{SATL}\right)}{2}\right) + \left(1 - \theta\right)V_R$$

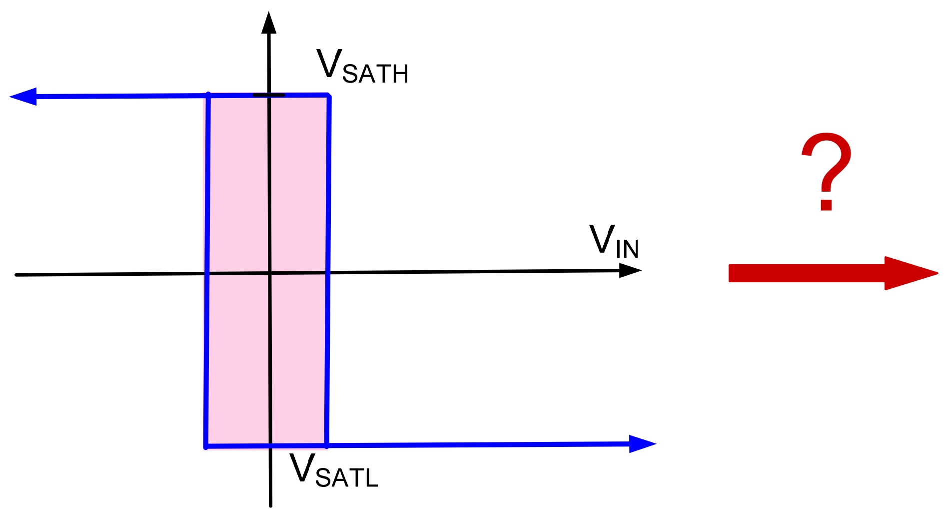
If $V_{SATH} = V_{DD}$, $V_{STAL} = V_{SS} = -V_{DD}$

$$V_W = 2\theta V_{DD}$$

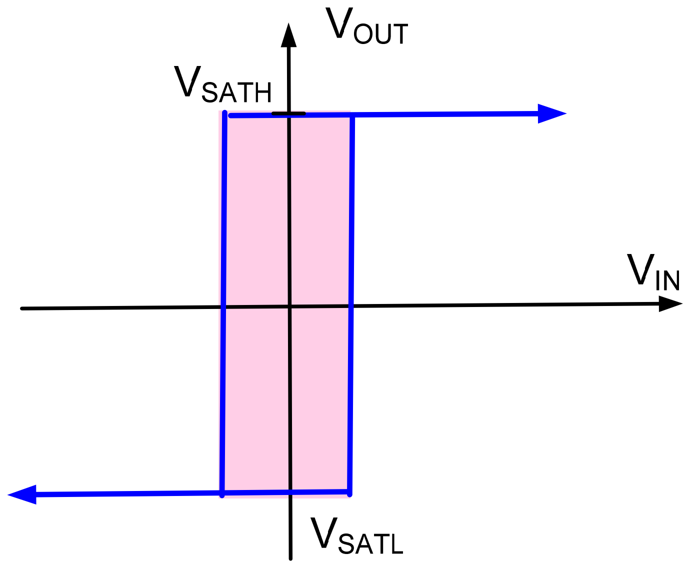
$$V_{CENT} = \left(1 - \theta\right)V_R$$

Shift can be to left or right depending upon sign of V_R

Inversion of Hysteresis Loop



Inverting Comparator with Hysteresis

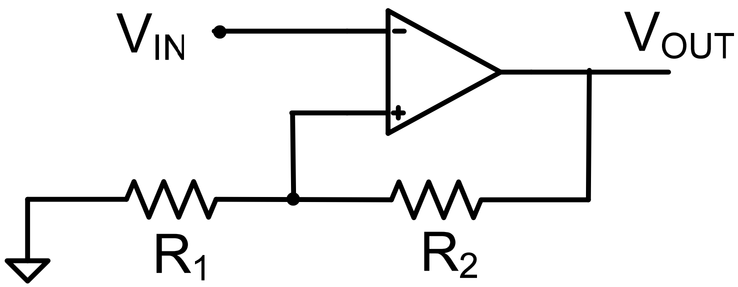


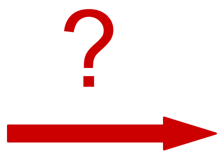
Noninverting Comparator with Hysteresis

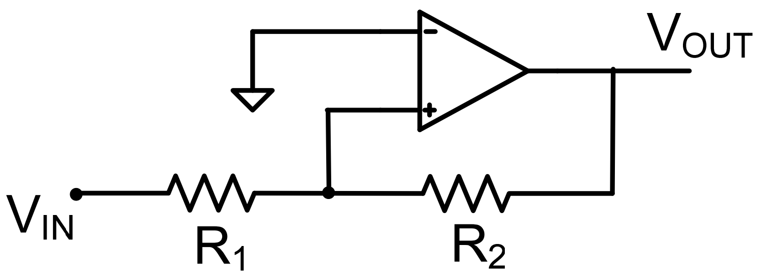
Strategies

- **Precede or follow inverting structure with an inverting amplifier**
- **Modify input location**

Inversion of Hysteresis Loop



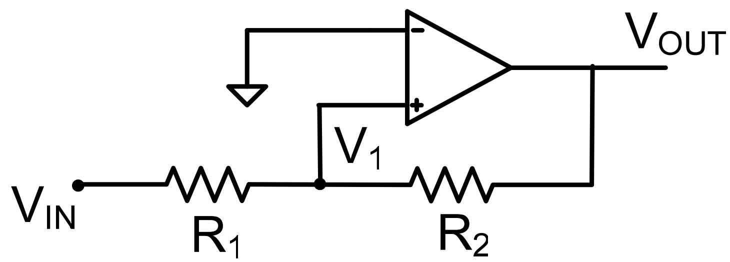




Inverting Comparator with Hysteresis

Noninverting Comparator with Hysteresis ?

Inversion of Hysteresis Loop



$$\theta_1 = \frac{R_1}{R_1 + R_2}$$

Noninverting Comparator with Hysteresis ?

Edges of hysteresis loop determined by condition where $V^+ = V^-$

For this circuit that is where

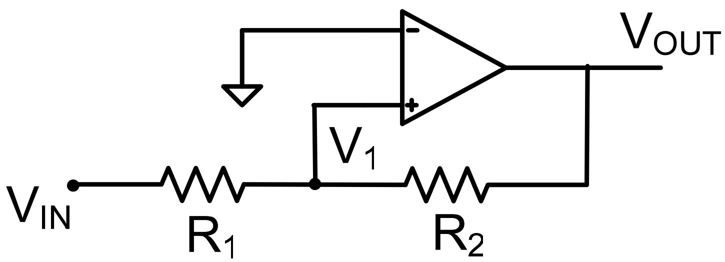
$$V_1 = 0$$

It follows from the 2-input voltage divider equation that

$$V_1 = \theta_1 V_{OUT} + \left(1 - \theta_1\right) V_{IN}$$

Substituting the second equation into the first, the edges of the hysteresis loop can be obtained by solving for the two possible values of V_{OUT}, V_{SATH} and V_{SATL}

Inversion of Hysteresis Loop



$$\theta_1 = \frac{R_1}{R_1 + R_2}$$

$$V_{SATH} \cong V_{DD} \qquad V_{SATL} \cong V_{SS}$$

Noninverting Comparator with Hysteresis ?

$$0 = \theta_1 V_{OUT} + (1-\theta_1) V_{IN} \longrightarrow \begin{cases} 0 = \theta_1 V_{SATH} + (1-\theta_1) V_{HYL} \\ 0 = \theta V_{SATL} + (1-\theta) V_{HYH} \end{cases}$$

Solving, we obtain

$$V_{HYH} = \frac{\theta_1}{\theta_1 - 1} V_{SATL}$$

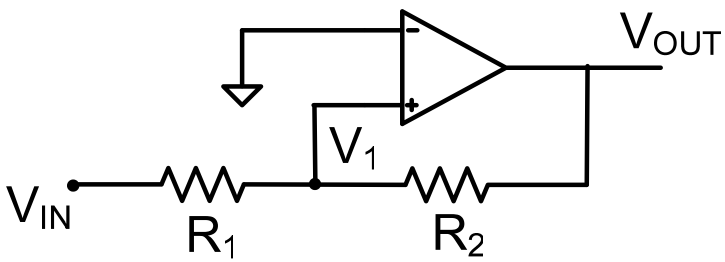
$$V_{HYL} = \frac{\theta_1}{\theta_1 - 1} V_{SATH}$$

If we define θ as $\theta = \frac{R_1}{R_2}$ we obtain

$$V_{HYH} = -\theta V_{SATL}$$

$$V_{HYL} = -\theta V_{SATH}$$

Inversion of Hysteresis Loop



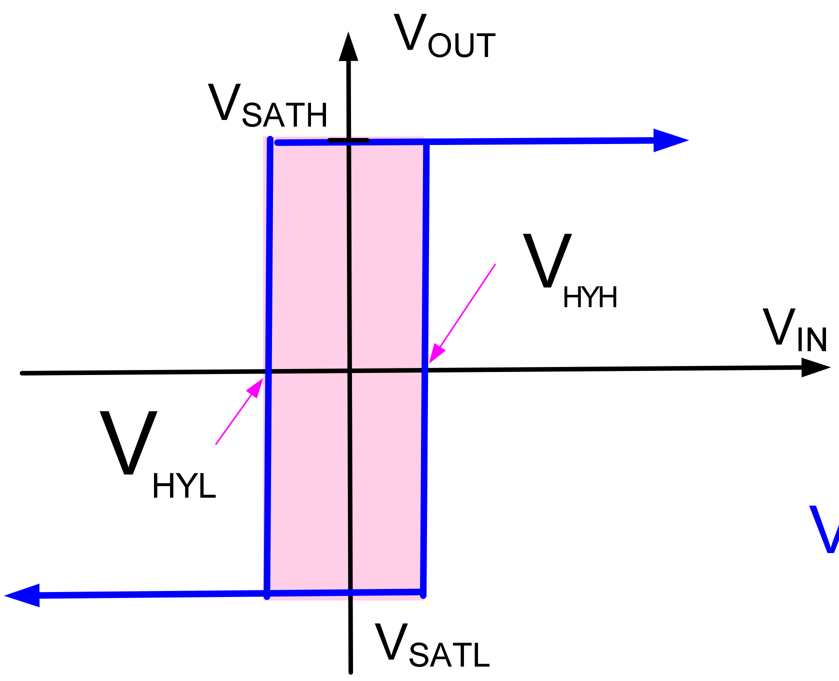
Noninverting Comparator with Hysteresis

$$\theta = \frac{R_1}{R_2}$$

$$V_{SATH} \cong V_{DD} \qquad V_{SATL} \cong V_{SS}$$

$$V_{HYH} = -\theta V_{SATL}$$

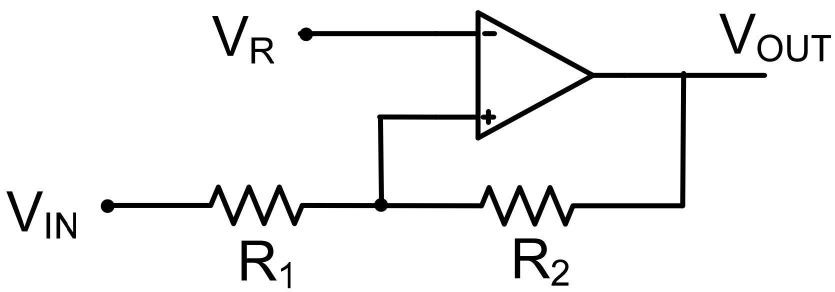
$$V_{HYL} = -\theta V_{SATH}$$



If $V_{SATH}=V_{DD}$, $V_{STAL}=V_{SS}=-V_{DD}$

$$V_{HYH} = \theta V_{DD} \qquad V_{HYL} = -\theta V_{DD}$$

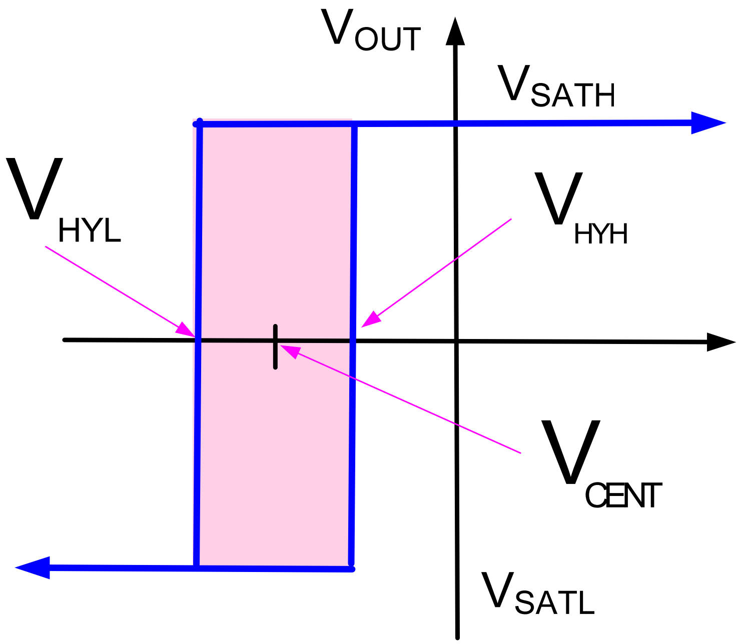
Shifted Inverted Hysteresis Loop



$$\theta = \frac{R_1}{R_2}$$

$$V_{SATH} \cong V_{DD} \qquad V_{SATL} \cong V_{SS}$$

Shifted Noninverting Comparator with Hysteresis



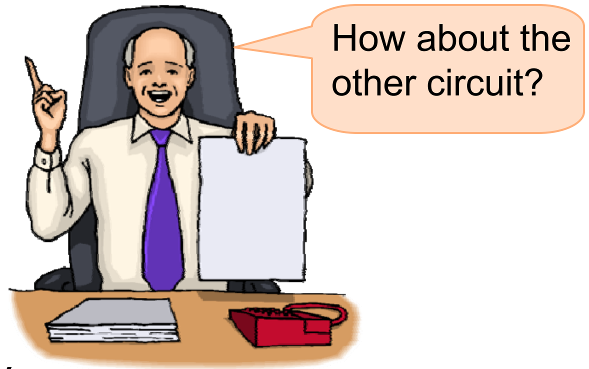
$$V_{HYH} = (1+\theta)\,V_R - \theta V_{SATL}$$

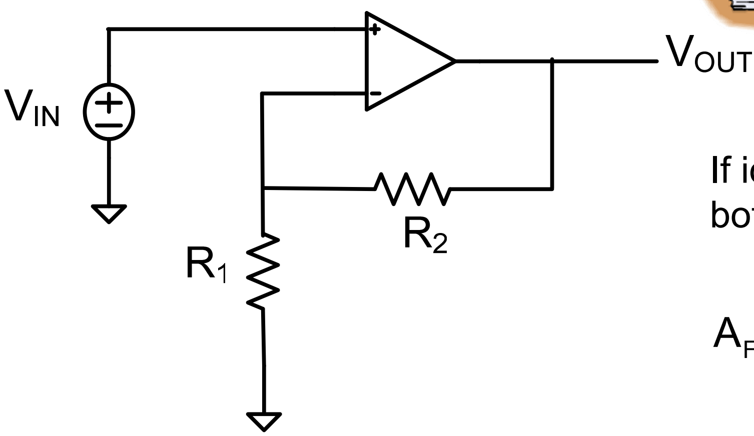
$$V_{HYL} = (1+\theta)\,V_R - \theta V_{SATH}$$

$$V_{CENT} = (1+\theta)\,V_R - \theta\left(\frac{V_{SATH} - V_{SATL}}{2}\right)$$

If $V_{SATH}=V_{DD}$, $\quad V_{STAL}=V_{SS}=-V_{DD}$

$$V_{HYH} = (1+\theta)\,V_R + \theta V_{DD} \qquad V_{HYL} = (1+\theta)\,V_R - \theta V_{DD}$$

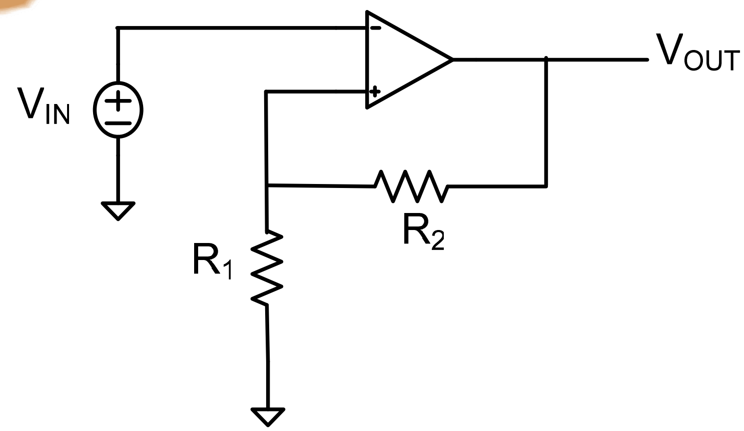




Usually the good circuit

If ideal op amps
both have gain

$$A_{FB}=1+\frac{R_2}{R_1}$$



Usually the bad circuit

This circuit is unstable !

Will now analyze the "usually good" circuit using nonlinear analysis method

End of Lecture 21